# ALONG-TRACK RESOLUTION ENHANCEMENT FOR WIDE-BANDWIDTH, LOW-FREQUENCY SAR BY ACCOUNTING FOR THE WAVELENGTH CHANGE OVER THE BANDWIDTH

*Evan C. Zaugg and David G. Long*

Brigham Young University Microwave Earth Remote Sensing Laboratory
459 Clyde Building, Provo, UT 84602 801-422-4884 zaugg@mers.byu.edu

### ABSTRACT

Common methods for processing SAR data, such as the Range-Doppler Algorithm (RDA) and the Chirp Scaling Algorithm (CSA), make certain approximations about the SAR signal. As the transmit frequency drops, the bandwidth grows, and the beamwidth increases, the approximations miss important factors required to precisely process the data. This paper shows that the approximations correspond to keeping lower order terms of the expansion of the SAR transfer function. We demonstrate the limits for focusing low frequency SAR data with these approximations. Previous methods for correcting the approximation errors are shown and a new method for including an arbitrary number of terms in processing the data is discussed. The concepts presented are verified using simulated SAR data.

## 1. INTRODUCTION

New synthetic aperture radar systems operating with wide bandwidths at low frequencies attract attention due to the potential for improving the data quality of existing applications and investigating new uses. At low frequencies the approximations made in formulating a number of SAR processing algorithms, such as the Range-Doppler Algorithm (RDA) and the Chirp Scaling Algorithm (CSA), are no longer good approximations. The errors caused by many of these approximations have been addressed, together with suggested remedies, in previous works. Also at low frequencies, a wider beamwidth is required for high azimuth resolution, this causes problems with the center beam approximation used in motion compensation [1] and in the chirp scaling process used in the CSA [2]. The wide bandwidth means that the wavelength changes appreciably over the radar chirp, but the approximations fail to fully account for the widely varying wavelength, resulting in azimuth de-focusing [3].

Processing for low frequency SAR's is typically done with the wavenumber domain Omega-K [4] algorithm or time domain processing methods [5]. These methods avoid the approximations that make RDA and CSA problematic at low frequencies and with wide beamwidths. Unfortunately these methods are much less computationally efficient. The $\omega$-k processing requires a costly interpolation to perform the Stolt mapping, and the algorithm makes it difficult to implement a range-dependent motion compensation method. Time domain methods can be very precise for all SAR configurations, but are very inefficient. Efforts have been made to modify the CSA to efficiently handle the effects of the wide aperture and varying wavelengths [1], to extend the $\omega$-k algorithm to handle motion compensation [4], and to streamline the time domain methods.

This paper explores the effects of the approximations made in SAR processing. In Section 2, the general SAR signal is derived and in Section 3, the approximations are analyzed. Simulated data is presented to demonstrate the effects of the approximations. Section 4 explains the previous efforts to efficiently handle the approximation errors, and Section 5 presents the possibility of a generalized algorithm that accounts for an arbitrary number of terms in the approximation is discussed.

## 2. THE GENERAL SAR SIGNAL

For our analysis, we consider only the phase functions of the SAR signal, ignoring the initial phase. As in the development presented in [6] we can describe the phase of the demodulated baseband SAR signal as

$$\Phi_0 = -4\pi f_0 R(\eta)/c + \pi K_r(\tau - 2R(\eta)/c)^2 \qquad (1)$$

where $f_0$ is the carrier frequency. $R(\eta)$ is the range to a given target at slow time $\eta$. $K_r$ is the chirp rate and $\tau$ is fast time.

The first term describes the azimuth modulation, it consists of the phase left over after demodulation. It is purely a function of the carrier frequency and the changing range to a target. The second term in Eq. (1) is the transmit chirp delayed by the two-way travel time to the target. If we were to reduce our bandwidth to a single frequency, the second term would go to zero, but we would still have the same azimuth modulation.

The approximations made in SAR processing algorithms are made to the signal in the wavenumber, or two-dimensional frequency domain. To obtain an expression for the signal in this domain, we take the range and azimuth Fourier transforms of Eq. (1). We approximate the Fourier transforms using the principle of stationary phase (POSP), which is valid expect in the extreme case of having radar frequencies near zero.

As shown in [6], the wavenumber domain expression is

$$\begin{aligned}\Phi_{1RA} &= -\frac{4\pi R_0(f_0+f_\tau)}{c}\sqrt{1-\frac{c^2 f_\eta^2}{4v^2(f_0+f_\tau)^2}} - \frac{\pi f_\tau^2}{K_r} \\ &= -\frac{4\pi R_0 f_0}{c}\sqrt{D^2(f_\eta)+\frac{2f_\tau}{f_0}+\frac{f_\tau^2}{f_0^2}} - \frac{\pi f_\tau^2}{K_r} \qquad (2)\end{aligned}$$

where

$$D(f_\eta) = \sqrt{1-\frac{c^2 f_\eta^2}{4v^2 f_0^2}}, \qquad (3)$$

$R_0$ is the range of closest approach, $v$ is velocity, $f_\tau$ is range frequency, and $f_\eta$ is azimuth frequency.

Eq. (2) is the phase of the SAR signal in the wavenumber domain. For a target at a given range $R_{ref}$, the target can be ideally focused with the reference function multiply

$$H_{RFM} = \frac{4\pi R_{ref} f_0}{c}\sqrt{D^2(f_\eta)+\frac{2f_\tau}{f_0}+\frac{f_\tau^2}{f_0^2}} + \frac{\pi f_\tau^2}{K_r} \qquad (4)$$

This works regardless of squint, beamwidth, and chirp bandwidth.

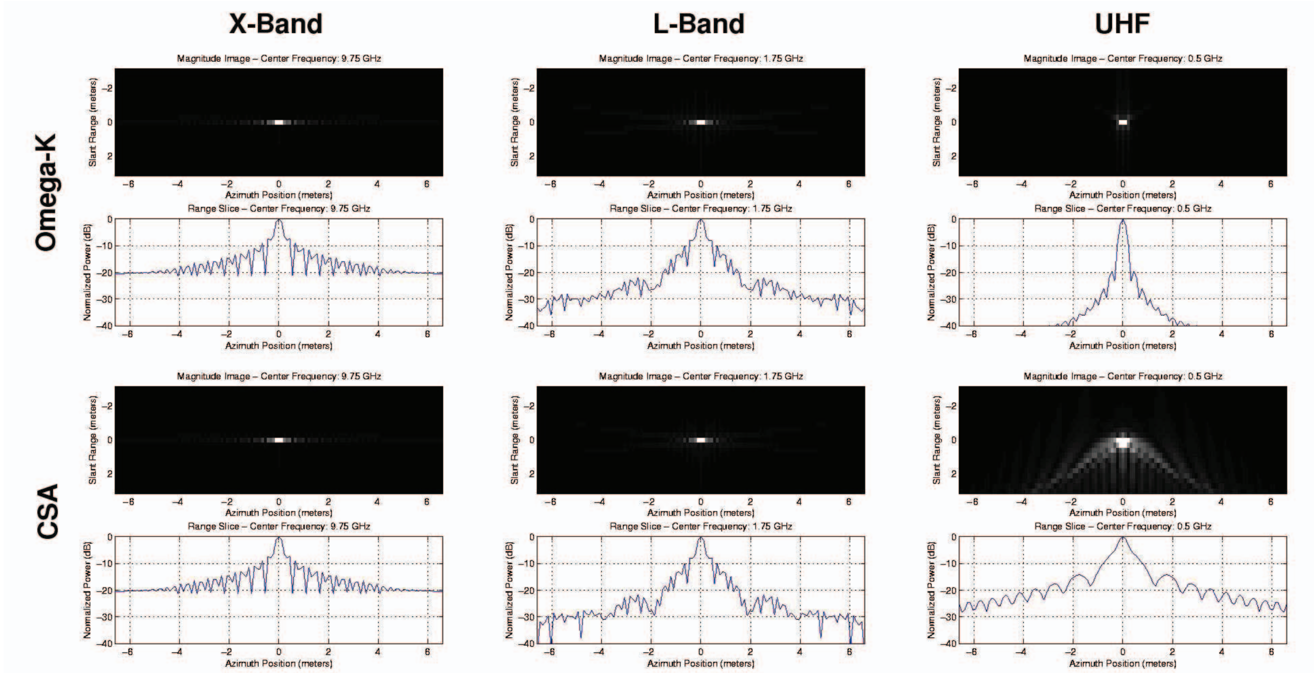


**Fig. 1**. 500 MHz bandwidth SAR data is simulated for a single point target at three frequencies with two processing algorithms. The columns from left to right are the center frequencies, 9.75 GHz, 1.75 GHz, and 500 MHz. The top row shows images processed with the Omega-K algorithm and at the bottom the same data is processed with the CSA. The simulation parameters are identical for each data set except for the beamwidth which varies with the center frequency to maintain the same theoretical azimuth resolution for each example. The measured azimuth resolutions obtained by these trials are compared in Table 1.

## 3. SAR APPROXIMATIONS

The Omega-K algorithm uses the exact representation of Eq. (2) and applies Eq. (4) for a reference range. Stolt interpolation is done to correct for the other ranges. This makes the $\omega$-K algorithm a good choice for systems with low-frequency, a large beamwidth, and a large bandwidth. This precision comes at the cost of high complexity and high processing time compared to the CSA and RDA.

Other algorithms use a Taylor series approximation of Eq. (2). The square root term can be expanded as

$$\begin{aligned}\Upsilon(f_\tau) &= \sqrt{D^2(f_\eta) + \frac{2f_\tau}{f_0} + \frac{f_\tau^2}{f_0^2}} \\ &\approx \Upsilon(0) + \frac{\Upsilon^{'}(0)}{1!} f_\tau + \frac{\Upsilon^{''}(0)}{2!} f_\tau^2 + \frac{\Upsilon^{'''}(0)}{3!} f_\tau^3 \ldots \end{aligned} \quad (5)$$

RDA keeps only the 0th order term

$$\Phi_{RDA} \approx -\frac{4\pi R_0 f_0}{c} \cdot [D(f_\eta)] - \frac{\pi f_\tau^2}{K_r} \quad (6)$$

which makes the algorithm relatively simple. The first term of Eq. (6) is the azimuth modulation, corrected in the range-Doppler domain during "azimuth compression." The second term is the chirp modulation corrected in the "range compression" step. The range-cell migration (RCM) correction is an interpolation that makes up for the neglected RCM term and the secondary range compression compensates for neglected higher order terms.

The CSA keeps up to the second order terms

**Table 1**. Simulated along-track resolution for a 500 MHz bandwidth SAR with theoretic along-track resolution of 24 cm, as show in Fig. 1. The beamwidth varies with the center frequency to maintain the same azimuth resolution for each test.

| | Center Frequency | | |
|---|---|---|---|
| Algorithm | 500 MHz | 1.75 GHz | 9.75 GHz |
| Omega-K | 0.243 m | 0.289 m | 0.294 m |
| CSA | 0.437 m | 0.290 m | 0.294 m |
| Azimuth Beamwidth | 77.30° | 20.56° | 3.67° |

$$\begin{aligned}\Phi_{CSA} &\approx -\pi f_\tau^2/K_r - 4\pi R_0 f_0/c \cdot \\ &\left[D(f_\eta) + \frac{f_\tau}{f_0 D(f_\eta)} + \frac{D^2(f_\eta)-1}{2f_0^2 D^3(f_\eta)} f_\tau^2\right]\end{aligned} \quad (7)$$

In the square brackets, the first term is the azimuth modulation, the second term is the range-cell migration, and the third term is "cross-coupling" between the range and azimuth frequencies.

More generally, we expand Eq. (5) to an arbitrary number of terms:

$$\begin{aligned}\Upsilon(f_\tau) &\approx D(f_\eta) + \frac{f_\tau}{f_0 D(f_\eta)} + \frac{D^2(f_\eta)-1}{2f_0^2 D^3(f_\eta)} f_\tau^2 \\ &- \frac{D^2(f_\eta)-1}{2f_0^3 D^5(f_\eta)} f_\tau^3 - \frac{5 - 6D^2(f_\eta) + D^4(f_\eta)}{8 f_0^4 D^7(f_\eta)} f_\tau^4 \ldots\end{aligned} \quad (8)$$

From this equation we can explore how the approximations effect the signal at different frequencies and bandwidths. Each term

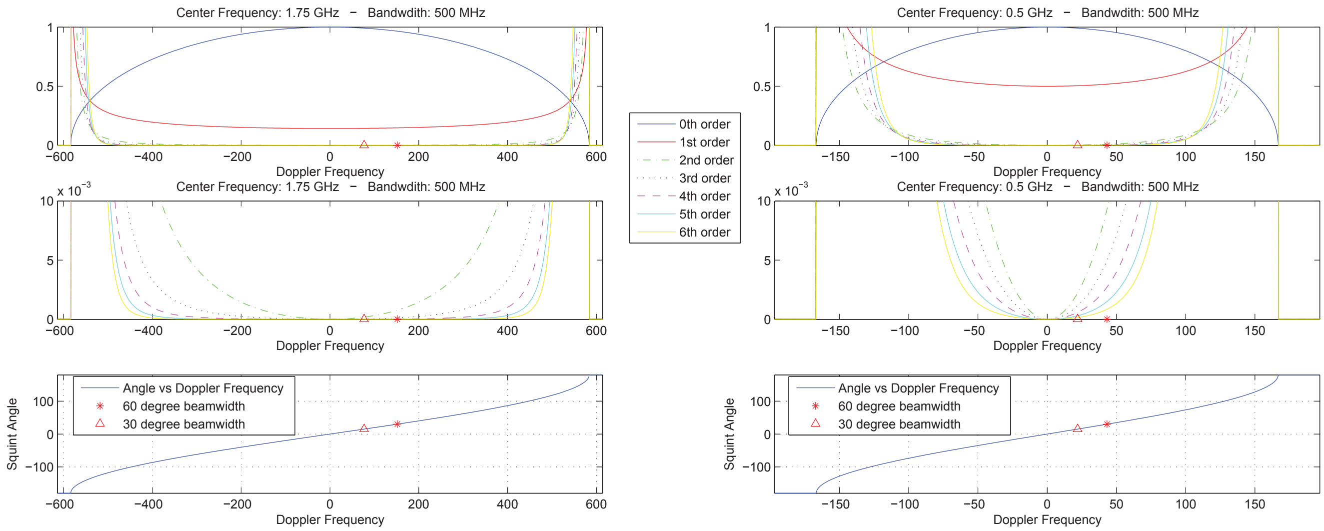


**Fig. 2**. The maximum magnitudes of the different order terms of Eq. (9) for a 500 MHz bandwidth at L-band (left) and UHF (right). The top row shows the orders 0-6 while the middle row focuses on just the terms of order 2-6. The bottom row shows the relationship between beamwidth and Doppler bandwidth for the velocity of 50 m/s used in the calculations. As the center frequency decreases, the higher order terms become more important at smaller beamwidths. In addition, at lower frequencies a much larger beamwidth is required to maintain the same azimuth resolution, making it doubly important to account for the higher order terms.

of order $n$ in this expansion has a $f_\tau^n/f_0^n$ term, which becomes more important as the change in wavelength over the chirp increases. When large enough, we must account for these higher order terms to properly focus the image.

Simulated data is used to evaluate the azimuth focusing with the different approximations for three different center frequencies (see Fig. 1). For purposes of comparison between different center frequencies, the simulation parameters are identical except for the antenna beamwidth which changes with the center frequency to maintain the same azimuth resolution. In the simulation, there is a single target at a known range. With the Omega-k algorithm, Eq. (4) is used to "perfectly" focus the target. This is compared to processing the same data with CSA approximations of Eq. (7). The measured azimuth resolutions are compared in Table 1. It is clear that for lower frequencies, higher bandwidths, and larger beamwidths, the neglected higher order terms of Eq. (9) become more important (see Fig. 2). In fact, Eq. (9) shows that the challenges of processing high-squint, large beamwidth, high bandwidth, and low frequency SAR all are manifestations of the same root cause: approximations that neglect the higher order terms.

## 4. PREVIOUS COMPENSATION METHODS

There have been a couple of attempts to efficiently address the issue of large changes in wavelength. In [1] a correction is suggested for a large change in wavelength using the CSA. After the azimuth matched filter is applied in the range-Doppler domain the data is transformed back into the wavenumber domain by separating the data into small blocks in range and performing a range FFT on each block. A corrective phase function is applied that corrects for the change in wavelength

$$\Delta H_{AZ}(f_\eta, f_\tau) = e^{\left(j4\pi R_b\left(\frac{D_\lambda(f_\eta,f_\tau)}{\lambda_\tau} - \frac{D(f_\eta)}{\lambda_0}\right)\right)} \tag{9}$$

where $\lambda_\tau$ is the wavelength corresponding to frequency $(f_0 + f_\tau)$, $R_b$ is range of the middle of the block being processed, and the $\lambda$-dependent range migration function is

$$D_\lambda(f_\eta, f_\tau) = \sqrt{1 - \left(\frac{\lambda_\tau f_\eta}{2V_r}\right)^2}. \tag{10}$$

While easy to implement, this method fails to address the true cause of the errors, and is thus unreliable. Simulations show that in certain situations it improves the focusing, while in other situations it makes things worse.

The second option is the Non-linear CSA algorithm proposed in [7] which keeps up to the 3rd order term of Eq. (9). Compared to the CSA, this method requires two additional range FFT's and a phase multiply. If terms higher than the 3rd order are needed for proper focusing, we would need something more than what the non-linear CSA can provide.

## 5. DEVELOPMENT OF NEW POSSIBILITIES

The goal then is to develop a new general processing scheme which efficiently accounts for as many higher order terms as dictated by the SAR parameters and the desired precision. Ideally, each additional term from Eq. (9) would add minimally to the computational burden.

A simple start is to apply a correction for higher order terms in the wavenumber domain for a given reference range. This can be done after an additional FFT at the beginning of processing, as in the non-linear CSA [7], or together with the bulk range cell migration correction in the traditional CSA. For example, a correction including up to the fourth order term

$$H_{4th} = -(4\pi R_{ref} f_0/c) \cdot \left( \frac{D^2(f_\eta) - 1}{2 f_0^3 D^5(f_\eta)} f_\tau^3 + \frac{5 - 6D^2(f_\eta) + D^4(f_\eta)}{8 f_0^4 D^7(f_\eta)} f_\tau^4 \right) \tag{11}$$

is applied to the data in the wavenumber domain. This only properly compensates for the higher order terms at the reference range, $R_{ref}$, but the difference with other ranges is small and may be neglected.

Applying this concept to the UHF data in Fig. 1, third-order and fifth-order compensations are used to generate the images in Fig. 3. The third-order image has an azimuth resolution improved by 16.2% over the CSA, while the fifth-order shows an improvement of 33.7%. These improvements are accomplished without any noticable increase in processing time.

Each term in the expansion of Eq. (9) is predictably computed from Eq. (5). Thus, a recursive step that applies corrections for an arbitrary number of terms can be implemented

$$H_{nth} = (4\pi R_{ref} f_0/c) \cdot \frac{\Upsilon^{(n)}(0)}{n!} f_\tau^n. \tag{12}$$

This is a key step in developing a generalized processing algorithm. Future development of this algorithm includes concrete determination of how many terms are required for proper focusing for specific radar parameters, corrections for the range dependence of the higher order terms (when they are not neglibable), and inclusion of a range-dependent velocity.

## 6. CONCLUSION

With new SAR systems pushing the frequencies lower, the bandwidths larger, and the beamwidths wider, the old approximations used in SAR algorithms miss important factors necessary to precisely process the data. This paper has shown how the higher order terms in the expansion of the SAR transfer function become more important for lower frequencies. Previous efforts to address this issue have been explained and the possibilities of a new generalized method have been developed. An efficient algorithm accounting for higher order terms can be used to increase the precision over existing algorithms or even be implemented in real time, thus extending to utility of wide-bandwidth, low-frequency SAR.

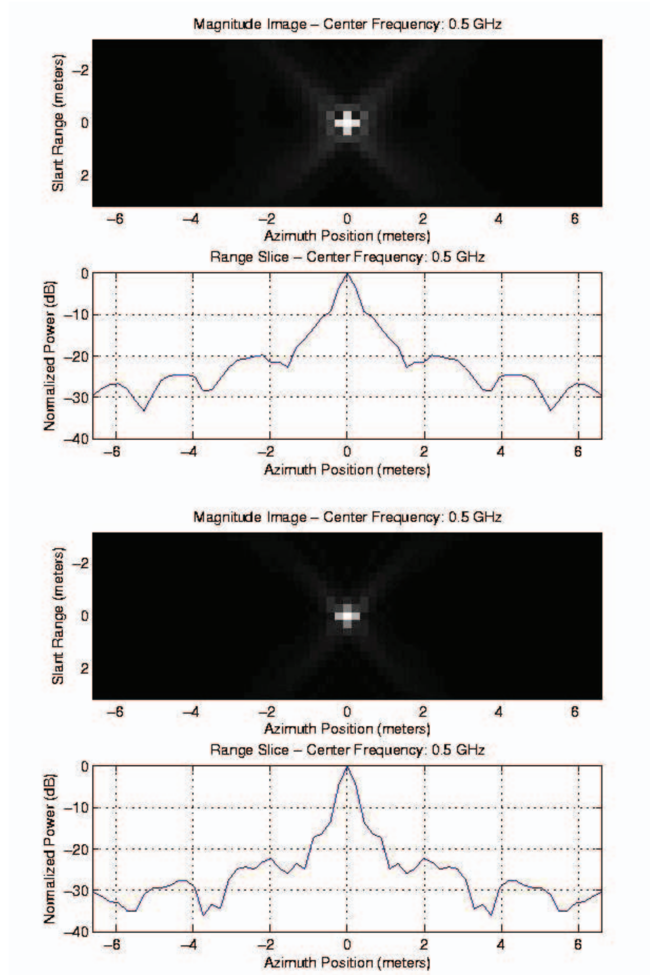


**Fig. 3**. The UHF data from Fig. 1 is reprocessed with the proposed method. The top image includes up to the third-order term and has a measured azimuth resolution of 0.3658 m. The bottom image was processed including up to the fifth-order term and has an azimuth resolution of 0.2896, which is comparable to the results from the Omega-K.

## 7. REFERENCES

[1] A. Potsis, A. Reigber, J. Mittermayer, A. Moreira,and N. Uzunoglou, "Improving the focusing properties of SAR processors for wide-band and wide-beam low frequency imaging," in *Proc. Int. Geosci. Rem. Sen. Symp.*, pp.3047-3049, vol.7, 2001.

[2] A. Potsis, A. Reigber, E. Alivizatos, A. Moreira, and N.K Uzunoglou. "Comparison of Chirp Scaling and Wavenumber Domain Algorithms for Airborne Low-Frequency SAR." In *Francesco Posa, editor, SAR Image Analysis, Modeling, and Techniques V*, vol 4883, pp. 25-36, March 2002.

[3] J.M. Swiger, "Resolution limits of ultra wideband synthetic aperture radar using a rectangular aperture for FFT processing," *IEEE Trans. Aerospace and Electronic Systems*, vol.30, no.3, pp.935-938, Jul 1994.

[4] A. Reigber, E. Alivizatos, A. Potsis, and A. Moreira. "Extended wavenumber-domain synthetic aperture radar focusing with integrated motion compensation." in *Proc. IEEE Radar, Sonar nd Navigation*, Vol. 153, Iss. 3, pp.301-310, June 2006.

[5] A.F. Yegulalp,"Fast backprojection algorithm for synthetic aperture radar," in *The Record of the 1999 IEEE Radar Conference*, pp.60-65, 1999.

[6] I.G. Cumming and F.H. Wong, *Digital Processing of Synthetic Aperture Radar Data*, Artech House, 2005.

[7] G.W. Davidson, I.G. Cumming, and M.R. Ito. "A Chirp Scaling Aproach for Processing Squint Mode SAR Data," in *IEEE Trans. on Aerospace and Electronic Systems*, 32 (1), pp. 121-133, Jan. 1996.