# RAIN AND WIND ESTIMATION FROM SEAWINDS IN HURRICANES AT ULTRA HIGH RESOLUTION

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## ABSTRACT

A Bayesian method for estimating wind and rain in hurricanes from SeaWinds at ultra-high resolution is developed. We use a hurricane model to generate prior distributions for the wind speed, wind direction, and rain rate. The rain prior is derived from data from the Tropical Rainfall Measuring Mission Precipitation Radar (TRMM-PR). The new method reduces the variability of the standard simultaneous wind and rain estimates while preserving meso-scale detail.

## 1. INTRODUCTION

As an alternative to 25-km wind retrieval, an ultra-high-resolution (UHR) SeaWinds vector wind product may be used. The UHR product is posted on a 2.5-km grid but has an effective resolution of about 5-7 km [1]. In principle, this higher resolution provides the necessary detail to study the meso-scale structure of entire hurricanes. However, there are four major limitations of using the UHR products for hurricanes. First, the Ku-band geophysical model function (GMF) which relates wind to backscatter ($\sigma^0$) is not well understood for extremely high wind speeds. Second, high rain rates are common in hurricanes, and rain modifies the Ku-band backscatter signal. Erroneous wind vectors are retrieved if the rain is not accounted for. Third, ambiguity selection using the conventional method (nudging with numerical weather predictions) often causes the eye center locations to be misplaced. Fourth, the UHR product has a higher noise level than the conventional 25-km product, which complicates ambiguity selection and produces estimates with higher variance.

One method that mitigates these effects uses a hurricane wind field model as a prior in a Bayesian estimation scheme [2]. Using a prior reduces the effect of rain on the wind estimates. However, where rain rates are large, this method tends to deweight the measurements and impose the model more heavily. The rain contamination issue can be ameliorated using a simultaneous wind and rain retrieval method at ultra-high resolution (UHRSWR) [3]. However, this further increases the variability of the wind estimates and does not deal with ambiguity selection issues. A potential solution is to combine a Bayesian estimation scheme with UHRSWR.

This paper develops a method for estimating wind and rain in hurricanes at ultra-high resolution from the SeaWinds scatterometer using a Bayesian approach. A wind and rain prior is employed which reduces the variability of the wind and rain estimates and simplifies ambiguity selection. The procedure is based on maximum aposteriori probability (MAP) estimation and uses a simple statistical hurricane wind and rain model to provide prior distributions that are used to modify the maximum likelihood (ML) objective function in the simultaneous wind and rain retrieval step. The low-order hurricane wind and rain prior is derived empirically from SeaWinds and TRMM-PR data.

## 2. METHOD

This section develops the theory and implementation of the new method—simultaneous wind and rain retrieval using MAP estimation (SWRMAP). MAP estimation for hurricane wind and rain retrieval is derived, the wind/rain field model for hurricanes used to generate the priors is developed, and the implementation for SeaWinds is described.

The new method employs MAP estimation to retrieve the wind/rain vector, denoted $\bar{U}$, from the reconstructed $\sigma^0$ measurement vector, denoted $\bar{\sigma}$, at each UHR (2.5 km) pixel within a hurricane. MAP estimation is a Bayesian approach that incorporates prior information with the measurements. The MAP estimator can be expressed as a slight modification of the ML estimator. For ML estimation we maximize the probability density function (PDF) of the measurements given the wind/rain vector $P(\bar{\sigma}|\bar{U})$. For UHR wind/rain retrieval, the measurements for each pixel are the reconstructed $\sigma^0$ fields for each of the four flavors of $\sigma^0$ measurements—vertical polarization fore- and aft-looking and horizontal polarization fore- and aft-looking [1]. Each flavor of UHR $\sigma^0$ is assumed to be statistically independent. Thus, the ML estimate for each pixel can be written as

$$\hat{\bar{U}}_{ML} = \underset{\bar{U}}{\text{argmax}} \left\{ \prod_i P(\sigma_i|\bar{U}) \right\}.$$

MAP estimation maximizes the PDF of the wind/rain vector given the measurements $P(\bar{U}|\bar{\sigma})$. Using Bayes rule this can be written as

$$P(\bar{\sigma}|\bar{U}) = \frac{P(\bar{\sigma}|\bar{U})P(\bar{U})}{P(\bar{\sigma})} = \frac{1}{P(\bar{\sigma})} \prod_i P(\sigma_i|\bar{U})P(\bar{U}).$$

Because $P(\bar{\sigma})$ is not a function of the wind/rain vector, the MAP estimator can be written as

$$\hat{\bar{U}}_{MAP} = \underset{\bar{U}}{\text{argmax}} \left\{ \prod_i P(\sigma_i|\bar{U})P(\bar{U}) \right\}.$$

Note that $P(\bar{\sigma}|\bar{U})$ may be a multi-modal function of the wind/rain vector, which gives rise to wind/rain ambiguities. However, the prior term tends to emphasize one particular maximum and suppress the others—generally resulting in a unique wind/rain vector estimate corresponding to the dominant mode.

There are many schemes for obtaining appropriate prior distributions. For example, one may choose a non-informative prior (constant or uniform distribution), which would cause the MAP estimation problem to reduce to maximum likelihood estimation. Alternatively, one may apply a maximum entropy prior subject to some constraint. Both the uniform prior and the maximum entropy prior are useful when it is desirable to minimize the amount of information that the prior imposes on the estimates. Empirical priors may also be applied. These can be derived from wind/rain data from any source or sensor. Empirical priors may be global or specific to certain types of storms. For example, in [2] a prior for winds in hurricanes is derived.

The question to be addressed is which prior is the best. According to convex Bayes theory, the set of prior distributions is a convex set. That is, if we have multiple viable priors, any convex combination of the priors is also a reasonable prior [4]. Thus we may combine any two priors that are optimum according to two different criteria to obtain a new prior that represents a trade-off between the criteria. For example, we may combine an empirical prior with a uniform prior in order to reduce the influence that the prior has on the estimate.

In this paper we use a convex combination of an empirical prior with a non-informative prior for wind and rain in hurricanes. For the wind prior, we use the prior developed in [2]. This prior varies with certain hurricane parameters: the eye center location, maximum speed, and mean flow vector. We also develop a rain prior to add to this model that is a function of the distance between the pixel and the eye center.

Since the empirical prior is a function of the hurricane model parameters, we estimate the hurricane model parameters using the spatial model that relates the parameters of the priors between pixels. We call this relationship between the parameters of the priors the field-wise wind/rain model. We estimate the parameters of the model using model-based ML estimation based on the actual slice $\sigma^0$ measurements (not the reconstructed $\bar{\sigma}$ field). Once the model parameters are estimated, the wind/rain field is produced. The wind/rain vector at each pixel is directly related to the parameters of the prior for each pixel. Thus, we can generate the priors for each pixel.

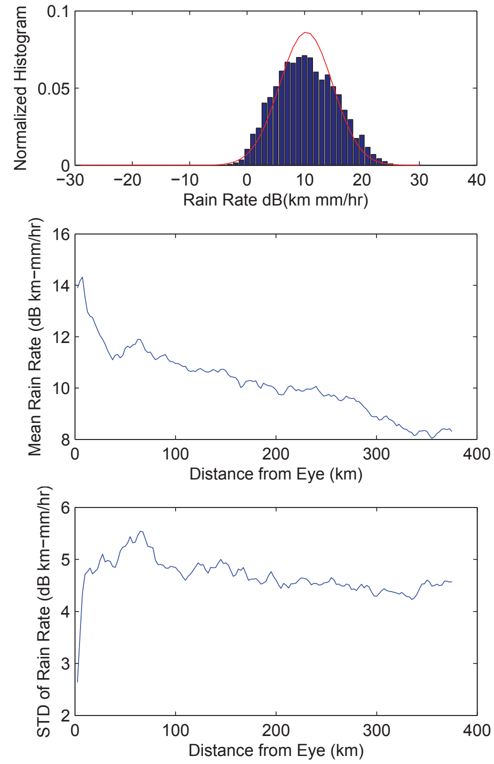


**Fig. 1**. TRMM-PR rain rate histogram with Gaussian fit superimposed (top), mean rain rate as a function of distance from the eye (middle), and standard deviation of the rain rate as a function of distance from the eye (bottom).

### 2.1. Wind/rain Model

We derive the model for the two-dimensional wind/rain field in a hurricane by using the same wind model developed in [2] and by deriving a simple model for the rain from TRMM-PR data. We restrict the rain model prior to be only a function of the distance from the eye center. Based on several different hurricanes we generate a histogram of all TRMM-PR rain rates greater than zero as a function of distance from the eye center. This produces a general prior for the rain rate given that it is raining $P(10\log_{10}(R)|R>0)$ as a function of the distance from the eye center.

Figure 1 shows the plots of the rain histogram for a particular distance from the eye, as well as the mean and standard deviation as a function of the distance from the eye. The histogram is similar to a Gaussian when the rain rate is expressed in dB. Thus, we assume that the prior is Gaussian in the log of the rain rate. We fit a line to the mean of the rain rate as a function of the distance from the eye and assume that the standard deviation is constant at 6 dB km-mm/hr. The form of the rain prior (given that it is raining) is thus

$$P(R_{dB}) = \frac{1}{\xi_R\sqrt{2\pi}} \exp\left(-\frac{(R_{dB}-\mu_R)^2}{2\xi_R^2}\right)$$

where $\mu_R$ and $\xi_R$ correspond to the mean and standard deviation of the distribution of the rain rate in dB.

### 2.2. Convex Bayes Priors

We combine a uniform prior with the empirical priors to enable control of the impact of the priors on the estimates. We introduce convex combination parameters for the wind speed prior $\alpha_s$, the wind direction prior $\alpha_d$, and the rain prior $\alpha_r$. The convex combination parameters determine how much we impose the empirical priors. Thus, we can increase the convex parameters to obtain a low variability at the expense of suppressing the small scale features or decrease the convex parameters to achieve the opposite. The convex rain prior is of the form

$$P_c(R_{dB}) = \alpha_r P_{Emp}(R_{dB}) + (1-\alpha_r)P_U(R_{dB})$$

where $P_{Emp}$ is the empirical prior, $P_U$ is the uniform prior, and $P_c$ is the convex prior. Similarly, the wind speed and direction priors can also be combined with a uniform prior. Note that the uniform prior is only constant over the search space (wind speed between 0 m/s and 50 m/s, wind direction between $0^o$ and $360^o$, and rain rates between -10 dB km-mm/hr and 22 dB km-mm/hr).

An information theoretic approach to choosing the convex parameters is considered. We may choose a prior that minimizes the Kullback-Leibler distance or the relative entropy between the ML and MAP probability density functions (PDFs) subject to a constraint on the variability of the estimates. The relative entropy between the MAP and ML PDFs is [5]

$$D(P(\bar{\sigma^0}|\bar{U})||P(\bar{U}|\bar{\sigma^0})) = -\int P(\bar{\sigma^0}|\bar{U})\log(P(\bar{U}))d\bar{U}$$

and represents the information added by imposing the prior. The variability of the estimate (first ambiguity) is related to the variance around the dominant peak of the MAP objective function. Thus, we may adjust the convex parameters closer to one to narrow the dominant peak due to adding more information from the prior. This method for determining the convex parameters is a function of the measurement geometry and the noise in the measurements, and is complicated to implement. For simplicity, in this paper we set the convex parameters to constants. We arbitrarily set the convex parameters $\alpha_s$, $\alpha_d$, and $\alpha_r$ to 0.2, 0.3, and 0.1, respectively.

### 2.3. Implementation

The first step in the new method is field-wise model-based estimation of the wind/rain field using the hurricane wind/rain field model. This produces estimates of the hurricane model parameters. Model-based maximum likelihood estimation searches for the model instance that maximizes the joint probability of observing the $\sigma^0$ slice measurements given that the model instance is the true wind/rain field. Thus, the estimate of the wind/rain field $\hat{\bar{U}}(x,y)$ is given by

$$\hat{\bar{U}}(x,y) = \underset{\bar{U}(x,y)=g(\bar{a})}{\operatorname{argmax}} \left\{ \prod_s P(\sigma_s^0 | \bar{U}(x,y)) \right\}$$

where $\bar{U}(x,y) = g(\bar{a})$ is the wind/rain field on an $(x,y)$ grid produced by the model $g(\bar{a})$ where $\bar{a}$ represents the model parameters. $P(\sigma_s^0|\bar{U}(x,y))$ is the PDF of a slice measurement $\sigma_s^0$ given the wind, which has the form

$$P(\sigma_s^0|\bar{U}(x,y)) = \frac{1}{\sqrt{2\pi}\xi_s} \exp \frac{-(\sigma_s^0 - \mathrm{gmf}_s(\bar{U}(x,y)))^2}{2\xi_s^2}$$

where $\xi_s^2$ is the variance and

$$\mathrm{gmf}_s(\bar{U}(x,y)) = \sum_x \sum_y A_s(x,y)\mathrm{gmf}_t(\bar{U}(x,y), \theta_s(x,y), \psi_s(x,y), pol, f)$$

where $\mathrm{gmf}_t$ is the true (high resolution) geophysical model function, $A_s(x,y)$ is the antenna gain pattern for the slice projected onto the Earth, $\theta_s$ is the incidence angle, $\psi_s$ is the azimuth angle, $pol$ is the polarity, and $f$ is the center frequency of the slice.

Once the hurricane model parameters are estimated, the priors for each pixel are computed and MAP estimation using the priors on the wind speed, wind direction, and rain rate is employed to estimate the wind/rain vector for each pixel using the UHR $\bar{\sigma}$ field. This produces multiple ambiguities similar to ML estimation. However, due to the inclusion of the prior, the first ambiguity (corresponding to the highest maximum) tends to have a likelihood value that is much higher than the others. Thus, we merely choose the first ambiguity as the final estimate and perform no further ambiguity selection. This provides an estimate of the wind/rain vector for each pixel given that it is raining.

In order to include non-raining cases, we perform wind-only retrieval using the MAP estimation scheme with priors on the speed and direction. The first ambiguity provides the best estimate (in the MAP objective function sense) of the wind given that it is not raining. To choose whether the wind-only or the SWR estimate is best, we compare the probabilities (MAP objective function values) weighted by the probability that it is raining. That is, we combine the wind-only and SWR ambiguities to a single set of ambiguities according to

$$P_{new}(\bar{U}_i) = \begin{cases} p(R=0)P_{UHR}(\bar{U}_i) & \text{if } i < 4 \\ p(R>0)P_{SWR}(\bar{U}_{i-4}) & \text{if } i > 4 \end{cases}$$

where $p(R=0)$ is the probability that it is raining and $p(R>0) = 1 - p(R=0)$, and the subscript $i$ indexes the ambiguity. Then we sort the new list of ambiguities by $P_{new}$. The final wind/rain estimate for each pixel becomes the first ambiguity of the combined ambiguity list. For this paper we set $p(R=0)$ to 0.43 based on the probability of false alarm $P_{fa}$ and the probability of missed detection $P_{md}$ of the rain.

## 3. ANALYSIS

We analyze the MAP estimates of the wind and rain by comparing them to co-located data from other sources. We compare the rain estimates from the new MAP method and the UHRSWR method to TRMM-PR data and the wind estimates to H*WIND fields.

To investigate the quality of the rain estimates we compare the SWR and SWRMAP rain rates to co-located TRMM-PR rain rates of several storms. Figure 2 shows a log-density plot of the log of the SWR and SWRMAP rain rates versus the log of TRMM-PR rain rates. There are many low SWR rain rates where the TRMM-PR rain rates are relatively large. This underestimation is corrected in the SWRMAP estimates; however, there is a slight bias of the low rain rates. Although the variability of the rain estimates is improved with the SWRMAP method, they may be improved further with bias correction.

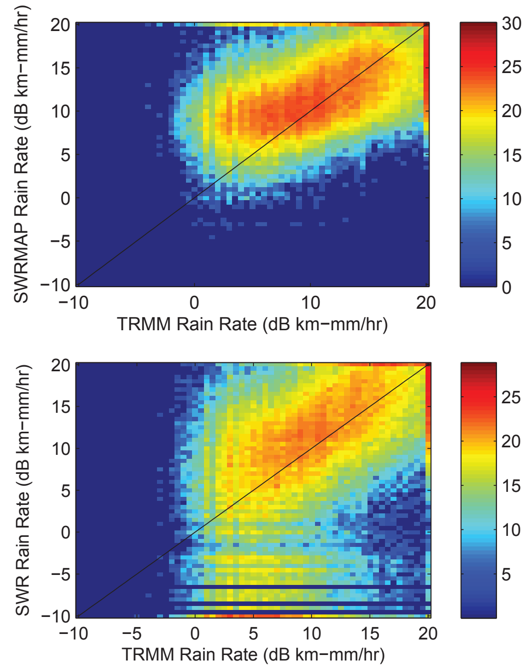


**Fig. 2**. Density plots of SWRMAP rain rate versus TRMM-PR rain rate (top) and SWR rain rate versus TRMM-PR rain rate (bottom).

It is difficult to validate the wind estimates since truth data is limited—we lack co-located wind data of similar temporal and spatial resolution to the UHR products. Nevertheless we compare the wind speed estimates to H*WIND products. Figure 3 shows the log-density plot of the SWRMAP and SWR wind speed estimates for a particular storm (Daniel 2000) in the Eastern Pacific basin. For H*WIND speeds less than about 15 m/s both methods produce similar speed estimates. However, the SWRMAP method reduces the variance of the estimates over the SWR method for H*WIND speeds higher than 15 m/s.

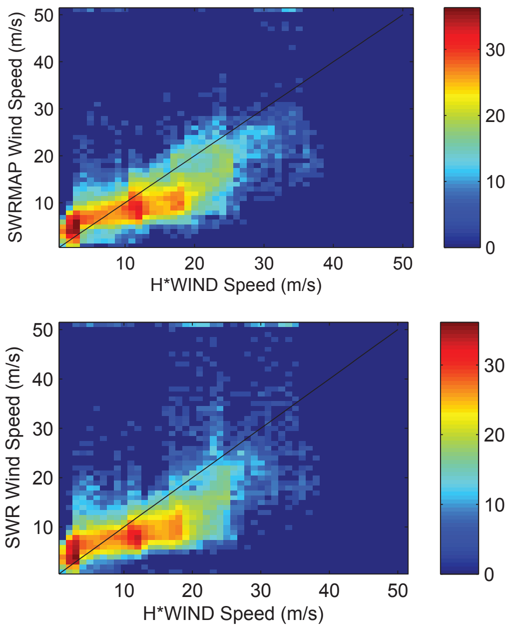


**Fig. 3**. Density plots of SWRMAP wind speed versus H*WIND speed (top) and SWR wind speed versus H*WIND speed (bottom).

Figure 4 displays the SWRMAP wind field, the SWR wind field, the SWRMAP rain field and a co-located TRMM-PR rain field for Hurricane Isaac on Sept. 29, 2000. The SWRMAP wind field is much less noisy than the SWR field and the SWRMAP winds in the rain-dominated regime (lower left quadrant) are closer to what is expected in a hurricane. Though noisy, the SWRMAP rain field has a similar spatial structure to the TRMM-PR rain field.

## 4. CONCLUSION

MAP estimation of hurricane wind and rain fields from the SeaWinds scatterometer is developed. The rain prior is derived from TRMM-PR data as a function of distance from the eye. The SWRMAP estimation method reduces the variability of the rain estimates and corrects the underestimation of low rain rates compared to the ML-based SWR method. The variability of the wind estimates is also reduced with the SWRMAP method.

## 5. REFERENCES

[1] D. G. Long, "High resolution wind retrieval from SeaWinds," *IGARSS*, pp. 751–753, 2002.

[2] B. A. Williams and D. G. Long, "Estimation of hurricane winds from SeaWinds at ultra high resolution," *Trans. on Geosc. and Remote Sensing*, 2008.

[3] D. G. Long, "Ultra high resolution rain retrieval from QuikSCAT data," *IGARSS*, pp. 4113–4116, 2006.

[4] W. C. Sterling and D. R. Morrell, "Convex bayes decision theory," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 21, no. 1, 1991.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, Wiley Interscience, 2005.

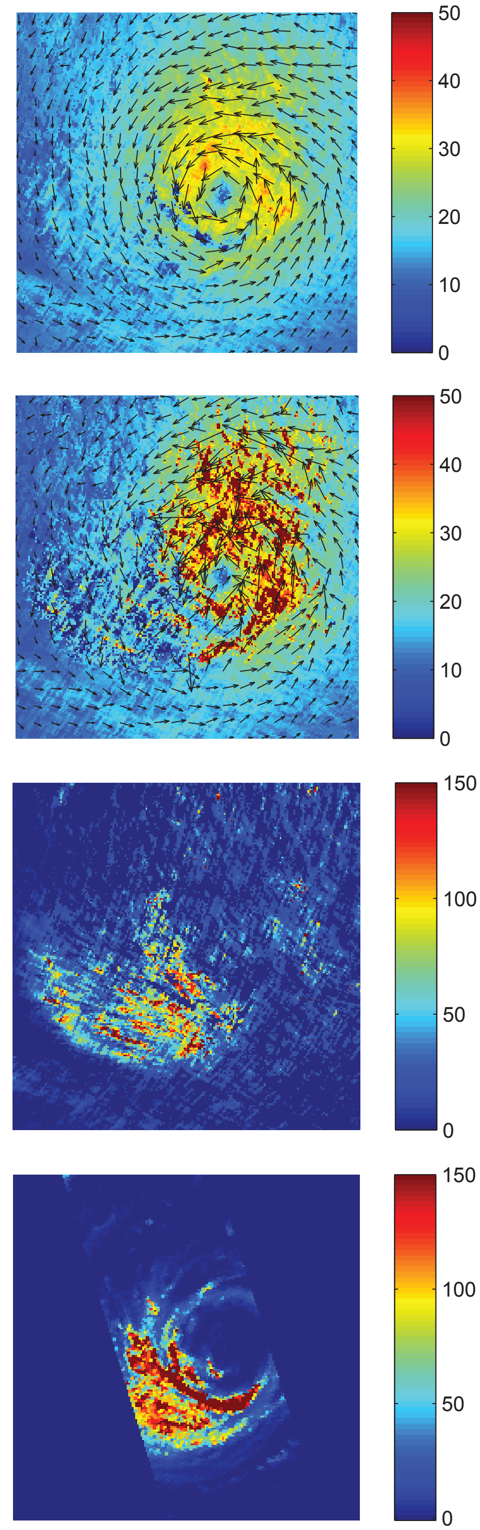

**Fig. 4**. SWRMAP wind field (top), SWR wind field (second), SWRMAP rain field (third), and co-located TRMM-PR rain rate (bottom). The colorbar is in m/s for the wind fields and km-mm/hr for the rain fields.