IDENTIFIABILITY IN WIND ESTIMATION FROM SCATTEROMETER MEASUREMENTS

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"Identifiability" is an important concept central to estimation theory. Loosely speaking, identifiability indicates whether or not an estimation procedure will yield a unique and consistent estimate of the desired parameters from the available measurements. We consider the problem of identifiability of a wind vector that is estimated from wind scatterometer measurements of the normalized radar backscatter (σ^o) of the ocean's surface at a given sample point using the geophysical model function relating the wind vector and σ^o.

In traditional wind estimation, an objective function is formulated using the noisy σ^o measurements and minimized to obtain estimates of the wind vector [1]. The objective function has several local minima with associated wind vectors termed *aliases* or *ambiguities*. Later processing, known as "de-aliasing" or "ambiguity removal," is used to select a single wind vector estimate. We provide a theoretical basis for this two-step approach by showing that the surface wind vector is *set-wise* or *system* identifiable; i.e., there is a unique *set* of wind vectors that could have given rise to the observed backscatter values and that the set of wind vector estimates is consistent. Within this set, there is no way to select a unique wind vector estimate from the measurements at a single sample point; hence, we establish a theoretical reason for dealiasing. Identifiability for the second dealiasing step is not addressed since dealiasing relies on information not present in the σ^o measurements at a single observation point.

Due to the nature of the geophysical model function (denoted by $\mathcal{M}$) and the fact that σ^o is observed from only a small set of azimuth angles, there may be several wind vectors which give rise to the same set of σ^o values (one σ^o value for each azimuth angle). Define the *true ambiguity set* D^c to be the set of all possible wind velocity vectors which give rise to the same set of $\sigma^o(k)$'s as the true wind vector $\mathbf{U}_t = (u_t, v_t)^T$, i.e.,

$$D^c \triangleq \left\{ \mathbf{U} \,\middle|\, \mathcal{M}\{\mathbf{U}, k\} = \mathcal{M}\{\mathbf{U}_t, k\} \;\forall\; k \right\}$$

where $\sigma^o(k) = \mathcal{M}\{\mathbf{U}, k\}$ is the σ^o corresponding to the k^{th} observation azimuth angle where the dependence of σ^o incidence angle and radar polarization are subsumed in the index k of $\mathcal{M}$.

Because members of the set D^c produce exactly the same set of $\sigma^o(k)$ values, they can not be distinguished from one another even if the measurements of σ^o are noise free. The membership in the set D^c depends on the model function, the set of relative azimuth angles (and the corresponding incidence angles and polarizations of the antenna beams), and the true wind vector. The fact that D^c may contain multiple members is a property inherent to the geophysical model function and the measurement geometry. The best we can ever expect to do is identify all members of D^c. Selection of a unique wind vector from D^c requires additional information not contained in the σ^o measurements for a single sample point of the ocean's surface; hence, the need for dealiasing in which data from other sample points (or, from other sources) are used in conjunction with dynamical constraints, continuity considerations, etc., to select a unique wind vector field.

For scatterometer wind estimation, the system model includes (a) the scatterometer measurement geometry (the number, polarizations, azimuth angles, and incidence angles of the measurements), (b) the geophysical model function, and (c) the σ^o noise measurement model. We assume that the geometric quantities and $\mathcal{M}$ are known. The set D^c is the set of parameter vectors for which the system model gives a perfect description of the true system. The scatterometer measurement noise is inversely proportional to measurement integration time T and is a quadratic function of the true σ^o [1,2]. A description of the scatterometer measurement system model is given in [2]. The maximum-likelihood (ML) estimator with objective function $J(\mathbf{U})$ is used to estimate the wind by minimizing $J(\mathbf{U})$ [1,2].

When D^c is multi-membered, the wind scatterometer system may be said to be *system identifiable* for a given estimation scheme iff

$$\lim_{T\to\infty} \inf_{\mathbf{U}\in D^c} ||\widehat{\mathbf{U}}(T) - \mathbf{U}|| = 0$$

where $\widehat{\mathbf{U}}(T)$ is the parameter vector estimate of $\mathbf{U}$. $\widehat{\mathbf{U}}(T)$ is a function of the σ^o measurements (which depend on T), the true system (including the true wind vector), the estimation scheme, and the system model. While the parameter estimate may not be unique for an identifiable system, the estimates will be consistent to within the set D^c.

To show identifiability of the point-wise estimation scheme using the maximum-likelihood approach [1], we need to show that as $T \to \infty$ [corresponding to a longer and longer measurement (for which the noise variance goes to zero)] the locations of global minima of $J(\mathbf{U})$ converge in probability to the members of D^c. To show convergence in probability we first show that: (A) $J(\mathbf{U})/T$ converges in the mean-squared sense (which is stronger than convergence in probability) to the deterministic function $E[J(\mathbf{U})/T]$ and (B) that the set of maximum-likelihood estimates of u and v converge in probability to the location of the minimum of $E[J(\mathbf{U})/T]$. Doing this is equivalent to showing that the set, A_p^c, of the $\mathbf{U}$ which minimize $J(\mathbf{U})/T$ for $T \to \infty$ is equal to D^c where A_p^c is defined as

$$A_p^c = \left\{ \mathbf{U}_1 \,\middle|\, \lim_{T\to\infty} E\left[\frac{J(\mathbf{U}_1)}{T}\right] = \min_{\mathbf{U}} \lim_{T\to\infty} E\left[\frac{J(\mathbf{U})}{T}\right] \right\}.$$

A_p^c is the set of maximum-likelihood estimates of $\mathbf{U}$ as $T \to \infty$. If $A_p^c = D^c$, the maximum-likelihood estimate is consistent and the set D^c is identifiable. Details of the proof are given in [3]. From this proof, it follows that wind vector estimation is therefore identifiable to the multimember set D^c. The wind estimate will be uniquely identifiable (i.e., to a single $\mathbf{U}$ estimate) if and only if D^c contains a single member.

Our results show that due to the nature of the geophysical model function, there will be a set of wind vectors which result from point-wise wind estimation, rather than a single estimate. In the limit, this set is unique and the vector estimates consistent. Within this set, selection of a single wind vector can not be made from the measurements taken at a single sample point.

References

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CH2825-8/90/0000-0553/$01.00