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Projecting an Arbitrary Latitude and Longitude onto a Tangent Plane

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1 Introduction

In working with points on the Earth's surface, it is convenient at times to work with points on defined on a locally tangent plane rather than in latitude and longitude. An example of such at time is in calculating the area of a region on the ground that is small enough that the curvature of the Earth has little effect. This report describes a simple algorithm for projecting an arbitrary latitude/longitude point onto a plane tangent to the Earth at an arbitrary point.

2 Projecting Latitude and Longitude to a Tangent Plane

A point at a given latitude and longitude, ϕ and θ respectively, can be projected to a unique point, (x,y), on a plane tangent to the surface of the earth at ϕ_0 and θ_0. The point of tangency is the center, (0,0), of the tangent plane. The orientation of the tangent plane is chosen such that the positive y-axis is North[1]. By way of definition, $\Delta\phi = \phi - \phi_0$, and $\Delta\theta = \theta - \theta_0$. The radius of the Earth is R_E. We can account for the oblateness of the Earth by calculating the local radius. The equation for calculating the local radius of the Earth is [1]

$$R_E = (1 - K_{flat} \sin^2(\phi)) R_a. \tag{1}$$

The related constants are found in Table 1. The radius of the local latitude line, ϕ, is R_ϕ.

[1] The point of tangency cannot be at a pole because at the poles North is not well defined.

Constant	Value	Description
K_{flat}	1/298.257	Earth flatness constant
R_a	6378.1363	Semi-major axis of the Earth (km)

Table 1: Earth radius constants [1]

Figure 1: Distances that need to be found in the conversion from latitude/longitude to x and y.

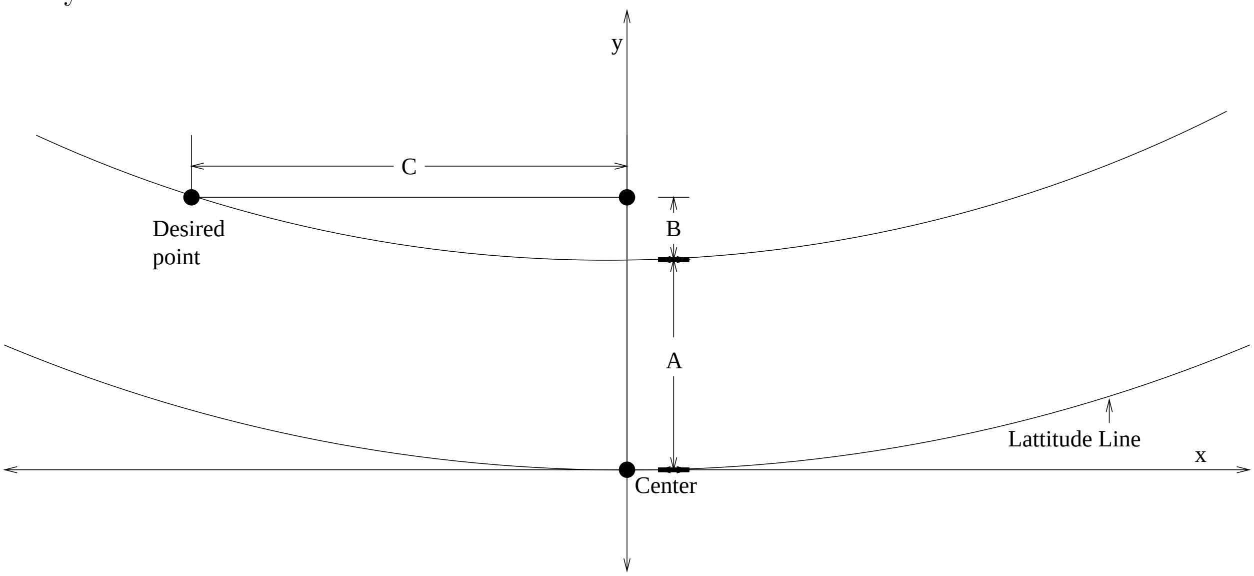

Figure 1 shows that there are three distances, A, B, and C, needed to calculate the conversion. The equations for the conversion are

$$R_\phi = R_E \cos(\phi), \tag{2}$$

$$A = R_E \sin(\Delta\phi), \tag{3}$$

$$B = R_\phi (1 - \cos(\Delta\theta)) \sin(\phi_0), \tag{4}$$

$$C = R_\phi \sin(\Delta\theta), \tag{5}$$

$$x = C, \tag{6}$$

and

$$y = A + B. \tag{7}$$

The units of x and y are the same as the units of the Earth's radius. (i.e. Using the Radius in Table 1 will give x and y in km.)

3 Conversion from a Tangent Grid to Latitude and Longitude

Reversing the projection has a couple of complications. First, the projection is not one to one, as each point on the plane has two corresponding points on the globe[2]. The equations given for conversion to latitude/longitude find the point on the same side of the globe as the

[2]This is for all points on the plane within the circle inscribed by the radius of the Earth. All points beyond this have no corresponding points on the globe. However there should not be any problems with this because the projection would not project any points beyond this circle.

tangent plane. Second, to reverse the projection, R_ϕ is required. However Eq. 2 shows that one of the unknowns, ϕ, is used in calculating R_ϕ. To overcome this, we can approximate $\phi \approx \phi_0$. If high accuracy is needed, it is possible to iterate this process replacing ϕ on each new iteration with that found in the last iteration.

The equations for the conversion are

$$\Delta\theta = \arcsin\left(\frac{x}{R_\phi}\right) \tag{8}$$

and

$$\Delta\phi = \arcsin\left(\frac{y - (1 - \cos(\Delta\theta))\sin(\phi_0) R_\phi}{R_E}\right). \tag{9}$$

The actual latitude and longitude are given by $\phi = \phi_0 + \Delta\phi$ and $\theta = \theta_0 + \Delta\theta$ where ϕ_0 and θ_0 are the latitude and longitude respectively of the point of tangency.

References

[1] David G. Long: *Xfactor7a* source code, Brigham Young University, 1998.